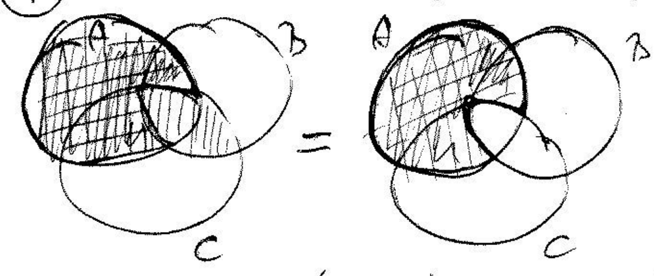
MAI X Y MARTIN KOLOŠ

(1) $A\setminus(B\cap C) = (A\setminus B)\cup(A\setminus C)$



PODLE VENNOVÝCH DIAGRAMŮ PLATÍ

PRO PRVEK Z X PLATÍ

LEVÁ STRANA

$x\in X \Leftrightarrow x\in (A\setminus(B\cap C)) \Leftrightarrow$

$x\in A \wedge x\notin (B\cap C) \Leftrightarrow x\in A \wedge \neg(x\in B\cap C)$

$\Leftrightarrow x\in A \wedge \neg(x\in B \wedge x\in C) \Leftrightarrow$

$\Leftrightarrow x\in A \wedge (x\notin B \vee x\notin C) \Leftrightarrow$

$\Leftrightarrow (x\in A \wedge x\notin B) \vee (x\in A \wedge x\notin C)$

$\neg(\alpha\wedge\beta) = \neg\alpha \vee \neg\beta$

$\alpha\wedge(\beta\vee\gamma) = (\alpha\wedge\beta)\vee(\alpha\wedge\gamma)$

PRO PRVEK Z MNOŽINY Y PLATÍ

(PS) PRAVÁ STRANA

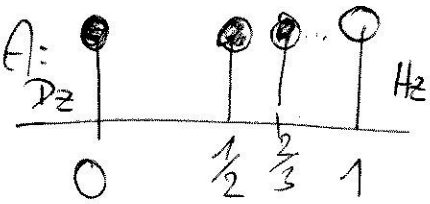
$x\in Y \Leftrightarrow x\in[(A\setminus B)\cup(A\setminus C)] \Leftrightarrow$

$\Leftrightarrow x\in(A\setminus B) \vee x\in(A\setminus C) \Leftrightarrow (x\in A\wedge x\notin B)\vee(x\in A \wedge x\notin C)$

MNOŽINY JSOU STEJNÉ – STEJNÁ PRAVIDLA PRO LIB. PRVEK

(2) $A = \{(1-\frac{1}{n}) \mid n\in\mathbb{N}\} = \{(\frac{n-1}{n}) \mid n\in\mathbb{N}\} = \{0; \frac{1}{2}; \frac{2}{3}; \frac{3}{4}; \dots\}$

$n=1: 1-\frac{1}{1} = 0$
$n=2: 1-\frac{1}{2} = \frac{1}{2}$
$n=3: 1-\frac{1}{3}$



$Hz = \langle 1; +\infty)$ HORNÍ

$Dz = (-\infty; 0\rangle$ DOLNÍ

ZÁVORY

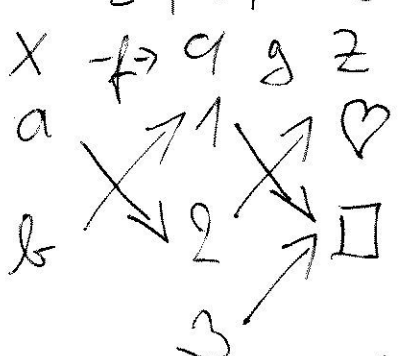
NEJMENŠÍ Z Hz JE 1 ⇒ $\sup A = 1$

NEJVĚTŠÍ Z Dz JE 0 ⇒ $\inf A = 0$

$1\notin A \Rightarrow \max A \nexists$

$0\in A \Rightarrow \min A = 0$

(3.) $X=\{a, b\}; Y=\{1,2,3\}; Z=\{\heartsuit; \square\}$



f JE ZOBRAZENÍ

g –||–

g∘f –||–

X g∘f Z

a → ♡

b → □

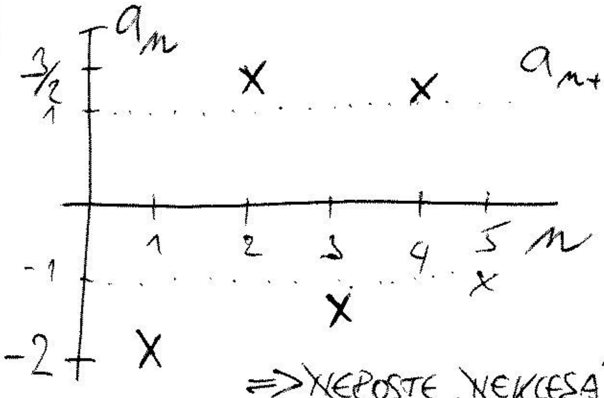
g∘f JE BIJEKCE

f JE INJEKTIVNÍ

g JE SURJEKTIVNÍ

4A (4.) $a_n = (-1)^n\left(\frac{n+1}{n}\right)$ KLESAJÍCÍ ? $a_{n+1} < a_n$

n	a_n
1	$(-1)^1\left(\frac{1+1}{1}\right) = -2$
2	$(-1)^2\left(\frac{2+1}{2}\right) = \frac{3}{2}$
3	$(-1)^3\left(\frac{3+1}{3}\right) = -\frac{4}{3}$
4	$(-1)^4\left(\frac{4+1}{4}\right) = \frac{5}{4}$
⋮	⋮



$a_{n+1} = (-1)^{n+1}\left(\frac{n+2}{n+1}\right)$

$\Rightarrow$ NEROSTE, NEKLESÁ, OMEZENÁ, DIVERGENTNÍ ↑

$$a = \lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^n\left(1+\frac{1}{n}\right) = \lim_{n\to\infty}(-1)^n \cdot \lim_{n\to\infty}\left(1+\frac{1}{n}\right) = \nexists \cdot 1 = \nexists$$

$a_{n+1} < a_n$ KLESAJÍCÍ PRO SUDÁ ČÍSLA

$$(-1)^{n+1}\left(\frac{n+2}{n+1}\right) < (-1)^n\left(\frac{n+1}{n}\right) \quad (n = 2k \text{ SUDÉ}) \quad (-1)^{2k+1}\left(\frac{2k+2}{2k+1}\right) < (-1)^{2k}\left(\frac{2k+1}{2k}\right)$$

~~$(-1)(2k)(2k+2)$~~

$$(-1)(2k)\cdot(2k+2) < (2k+1)(2k+1)$$

$$-4k^2 - 4k < 4k^2 + 4k + 1$$

$$0 < 8k^2 + 8k + 1 \quad ✓ \quad k > 0$$

$a_{n+1} > a_n$ ROSTOUCÍ PRO LICHÁ

$$(-1)^{2k+2}\left(\frac{2k+3}{2k+2}\right) > (-1)^{2k+1}\left(\frac{2k+2}{2k+1}\right) \qquad (2k+3)(2k+1) > \ominus(2k+2)(2k+2)$$

$$(-1)^2\left(\frac{2k+3}{2k+2}\right) > (-1)\left(\frac{2k+2}{2k+1}\right) \qquad 4k^2+6k+2k+3 > \ominus 4k^2 - 4k - 4$$

$$8k^2 + 12k + 7 > 0 \quad ✓$$

(5.) $\lim_{n\to\infty}\left(\sqrt{n-1} - \sqrt{n}\right) \overset{\text{DOSADÍME}}{=} \infty = |\infty - \infty|$

$$= \lim_{n\to\infty}\left(\sqrt{n-1} - \sqrt{n}\right)\frac{\sqrt{n-1}+\sqrt{n}}{\sqrt{n-1}+\sqrt{n}} = \lim_{n\to\infty}\left(\frac{n-1-n}{\sqrt{n-1}+\sqrt{n}}\right) = \frac{-1}{\infty+\infty} = 0$$

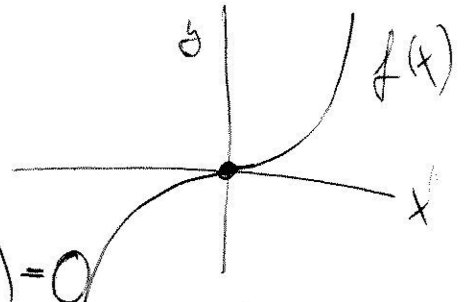
„KOUZELNÁ JEDNIČKA"

(8.) $f(x) = 2x\cdot\cos^2(x) + \frac{1}{x^2-1} = 2x\cdot\cos^2(x) + (x^2-1)^{-1}$

$$f'(x) = 2\cdot\cos^2(x) + 2x\cdot 2\cos(x)\cdot(-\sin(x)) + (-1)(x^2-1)^{-2}\cdot 2x \quad /$$

4IA I ⑥ $f(x) = \begin{cases} -x^2 & x \in (-\infty; 0) \\ +x^2 & x \in \langle 0; +\infty) \end{cases}$

POTENCIÁLNÍ BOD. NESPOJITOSTI JE 0



$\lim_{x \to 0} f(x) = f(0)$

f(x) SPOJITÁ JE

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x^2) = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2 = 0$

$f(0) = 0$

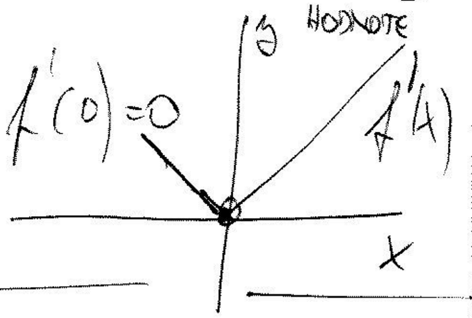
LIMITA ∃ A JE ROVNÁ FUNKČNÍ HODNOTĚ

$f'(x) = \begin{cases} -2x \\ 2x \end{cases}$

$\lim_{x \to 0^-} f'(x) = \lim_{x \to 0^-} (-2x) = 0$

$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^-} (2x) = 0$

$f'(0) = 0$



f'(x) JE SPOJITÁ

FUNKCE f(x) JE SPOJITÁ A MÁ I SPOJ. 1. DERIVACI

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⑦ $f'(x) = \lim_{\Delta \to 0} \frac{f(x+\Delta) - f(x)}{\Delta} =$ $\quad f(x) = \frac{1}{x}$ $\quad f(x+\Delta) = \frac{1}{x+\Delta}$

$= \lim_{\Delta \to 0} \frac{\frac{1}{x+\Delta} - \frac{1}{x}}{\Delta} = l \frac{\frac{x-(x+\Delta)}{(x+\Delta)x}}{\Delta} = l \frac{x - x - \Delta}{(x+\Delta)x \cdot \Delta} = \lim_{\Delta \to 0} \frac{-\Delta}{(x+\Delta)x \cdot \Delta} =$

$= \lim_{\Delta \to 0} \frac{-1}{(x+\Delta) \cdot x} = \frac{-1}{x^2}$, $\quad f(x) = \frac{1}{x}$ i $f'(x) = \frac{-1}{x^2}$

⑨ $x_0 = \pi/2$

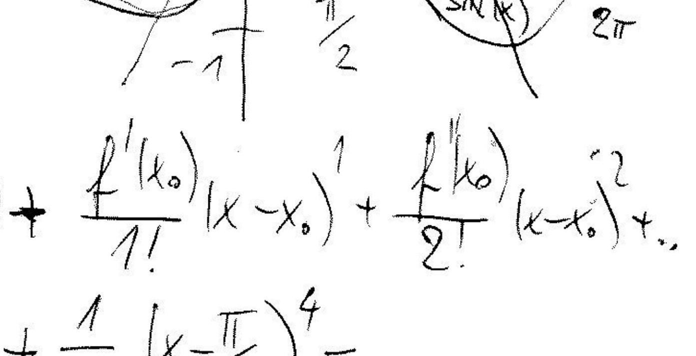
$f(x) = \sin(x)$ $\quad f(x_0) = \sin(\frac{\pi}{2}) = 1$

$f'(x) = \cos(x)$ $\quad f'(x_0) = \cos(\frac{\pi}{2}) = 0$

$f''(x) = -\sin(x)$ $\quad f''(x_0) = -\sin(\frac{\pi}{2}) = -1$

$f'''(x) = -\cos(x)$ $\quad f'''(x_0) = -\cos(\frac{\pi}{2}) = 0$

$f^{IV}(x) = \sin(x)$ $\quad f^{IV}(x_0) = \sin(\frac{\pi}{2}) = 1$



$T_k(x) = \sum_{n=0}^{k} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0)^1 + \frac{f''(x_0)}{2!}(x - x_0)^2 + \dots$

$= 1 + 0 - \frac{1}{2!}(x - \frac{\pi}{2})^2 + 0 + \frac{1}{4!}(x - \frac{\pi}{2})^4 - \dots$

$T_2(0) = 1 - \frac{1}{2}\frac{\pi^2}{4} = \frac{8 - \pi^2}{8} < 0$