MARTIN KOLOŠ
MAT. 3

(1)

$z_1 = 2i - 1 = -1 + 2i$ $\qquad z_1 + z_2 = 2i - 1 + 1 - i = 0 + i$

$z_2 = 1 - i$ $\qquad z_2 - z_1 = 1 - i - 2i + 1 = 2 - 3i$

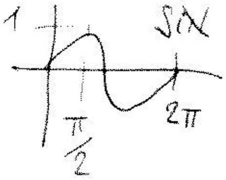
$$\boxed{i = \sqrt{-1} \qquad i^2 = -1}$$

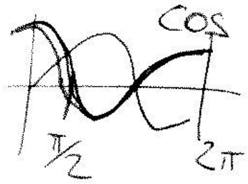
$z_1 \cdot z_2 = (2i - 1)(1 - i) = (\cancel{2i} - \underbrace{2i^2}_{+2} - 1 + i) = 1 + 3i$

$\frac{z_2}{z_1} = \frac{1-i}{2i-1} \cdot \underbrace{\frac{2i+1}{2i+1}}_{=1} = \frac{2i - 2i^2 + 1 - i}{4i^2 - 2i + 2i - \cancel{1}} = \frac{i+3}{-4-1} = -\frac{3}{5} - \frac{1}{5} i$

KOUZELNÁ JEDNIČKA

(2) $z = i = 0 + 1 \cdot i$





$a = 0$

$b = 1$ $\qquad r = |z| = \sqrt{0^2 + 1^2} = 1$

$\sin\varphi = \frac{b}{r} = 1$ $\qquad \varphi = \pi/2$ $\qquad z = i = 1 \cdot e^{i\pi/2}$ $\qquad \boxed{z = re^{i\varphi}}$

$\cos\varphi = \frac{a}{r} = 0$ $\qquad z^2 = \left(e^{i\frac{\pi}{2}}\right)^2 = e^{2i\frac{\pi}{2}} = e^{i\pi} = \cos\pi + i\sin\pi$

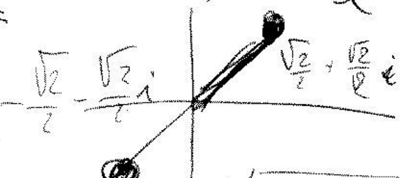
$z = a + bi = r(\cos\varphi + i \cdot \sin\varphi)$ $\qquad = -1 + 0$

$z_0^{1/2} = \left(e^{i\frac{\pi}{2}}\right)^{1/2} = e^{i\frac{\pi}{4}} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$

$z = e^{i\left(\frac{\pi}{2} + 2k\pi\right)}$ $\qquad k = 0$ $\qquad e^{i\frac{\pi}{2}} = z$

$k = 1$ $\qquad z = e^{i\left(\frac{\pi}{2} + 2\pi\right)}$ $\qquad z_1^{1/2} = \left(e^{i\left(\frac{\pi}{2} + 2\pi\right)}\right)^{1/2} = e^{i\left(\frac{\pi}{4} + \pi\right)}$

$= \; e^{\frac{4\pi}{5} i} = \cos\left(\frac{4}{5}\pi\right) + i\sin\left(\frac{4}{5}\pi\right) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i =$



(3.) $x^4 - 16 = 0$ $\qquad 2 \cdot 2 \cdot 2 \cdot 2 = 16 = 2^4$

$x^4 = 16$

$z = 16 = a + bi = 16 \cdot e^{i(0 + 2k\pi)}$

$\sqrt[4]{z} = |k = 0| = \left(16 \cdot e^{i \cdot 0}\right)^{1/4} = 2$

$|k = 1| = \left(16 \cdot e^{i(0 + 2\pi)}\right)^{1/4} = 2 \cdot e^{\frac{2\pi}{4}} = 2 \cdot e^{\pi/2} = 2i$

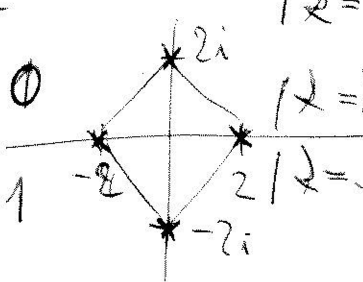
$|k = 2| = \left(16 \cdot e^{i(4\pi)}\right)^{1/4} = 2 \cdot e^{i\pi} = -2$

$|k = 3| = \left(16 \cdot e^{i(6\pi)}\right)^{1/4} = 2 \cdot e^{i\frac{3}{2}\pi} = -2i$

$a = 16$ $\qquad \sin\varphi = \frac{b}{r} = 0$

$b = 0$ $\qquad \cos\varphi = \frac{a}{r} = 1$

$r = 16$ $\qquad \varphi = 0$



ПАТ 3.

(4.) $P_3(x) = x^3 - 6x^2 + 11x - 6 = 0$

$P_3(x) = (x-1)P_2(x) = 0$

$P_2(x) = \frac{P_3(x)}{(x-1)} = ?$

$(x^3 - 6x^2 + 11x - 6) : (x-1) = x^2 - 5x + 6 = P_2(x)$

$-(x^3 - x^2)$

$-5x^2 + 11x - 6$

$-(-5x^2 + 5x)$

$6x - 6$

$-(6x - 6)$

0

$(x^2 - 5x + 6)(x-1) = 0$

$x^2 - 5x + 6 = 0 \qquad x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{5+1}{2} = 3, \quad \frac{5-1}{2} = 2$

$a = 1$

$b = -5 \qquad D = b^2 - 4ac = 25 - 4 \cdot 6 = 1$

$c = 6$

$x_0 = 1$

$x_1 = 2$

$x_2 = 3$

$x^2 - 5x + 6 = (x-3)(x-2) = x^2 - 2x - 3x + 6$

$(x-1)(x-2)(x-3) = 0$

(5.) $\frac{x^2+1}{x^2-1} = 1 + \frac{2}{x^2-1} \; (=)$

$(x^2+1) : (x^2-1) = 1 + \frac{2}{x^2-1}$

$-(x^2-1)$

2

$1 + \frac{2}{x^2-1} = \frac{x^2-1+2}{x^2-1} = \frac{x^2+1}{x^2-1}$

$x^2 - 1 = 0 \qquad x^2 - 1 = (x-1)(x+1) = 0$

$x^2 = 1 \qquad x_1 = -1$

$x = \pm\sqrt{1} \qquad x_2 = +1$

$(=) \; 1 + \frac{A}{x-1} + \frac{B}{x+1} = 1 + \frac{A(x+1) + B(x-1)}{(x-1)(x+1)} =$

$= 1 + \frac{Ax + A + Bx - B}{x^2 - 1}$

$2 = Ax + A + Bx - B$

$0 \cdot x + 2 = Ax + A + Bx - B \Rightarrow 0 = A + B, \quad 2 = A - B$

$A = -B \; ; \; B = -A$

$2 = A + A \qquad 2 = 2A$

$A = 1 \; ; \; B = -1$

$\frac{x^2+1}{x^2-1} = 1 + \frac{2}{x^2-1} = 1 + \frac{1}{x-1} - \frac{1}{x+1}$

6. $\sum_{n=1}^{\infty} a_n$; $a_n = \dfrac{n-1}{2^n(n+2)}$; $a_{n+1} = \dfrac{n+1-1}{2^{n+1}(n+1+2)}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\dfrac{n}{2^{n+1}(n+3)}}{\dfrac{n-1}{2^n(n+2)}} = \lim_{n\to\infty} \frac{n}{2^{n+1}(n+3)} \cdot \frac{2^n(n+2)}{n-1} =$$

$$= \lim_{n\to\infty} \frac{n(n+2)}{(n+3)(n-1)} \cdot \frac{\cancel{2^n}}{\cancel{2^n}\cdot 2} = \lim_{n\to\infty} \frac{n^2+2n}{n^2-n+3n-3}\,\frac{1}{2} = \lim_{n\to\infty} \quad \frac{1}{n^2} \Big/ \frac{1}{n^2}$$

$$= \lim_{n\to\infty} \frac{1+\frac{2}{n}}{1+\frac{2}{n}-\frac{3}{n^2}} \cdot \frac{1}{2} = \frac{1+0}{1+0-0} \cdot \frac{1}{2} = \frac{1}{2} < 1 \Rightarrow \text{KONVERGIRA}$$

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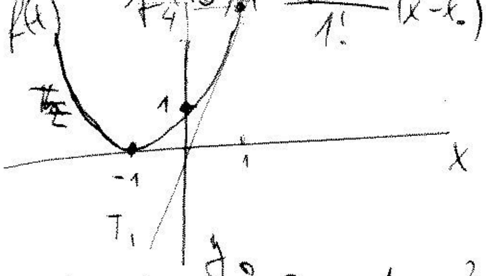
7. $f(x) = (x+1)^2$ $\qquad x_0 = 1$, $f(1) = 2^2 = 4$ $\qquad T_2(x) = \sum_{n=0}^{2} \dfrac{f^{(n)}(x_0)}{n!}(x-x_0)^n$

$f'(x) = 2(x+1)\cdot 1$ $\qquad f'(1) = 4$ $\qquad = \underset{4}{f(x_0)} + \dfrac{f'(x_0)}{1!}\overset{T_0}{(x-x_0)} + \dfrac{f''(x_0)}{2!}(x-x_0)^2 + \dots$

$f''(x) = 2$ $\qquad f''(1) = 2$

$f'''(x) = 0$ $\qquad f'''(1) = 0$



$$T_2(x) = 4 + 4(x-1) + \frac{2}{2}(x-1)^2 + 0 = 4 + 4x - 4 + x^2 - 2x + 1 = x^2 + 2x + 1$$

$$= (x+1)^2 = f(x) \quad (\text{JE TO POLINOM!})$$

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8. $$\int \left[\frac{x^2}{x}(x-1)^2 + x\cdot\sin(x^2)\right]dx = \int x\cdot(x^2-2x+1)\,dx + \int x\cdot\sin(x^2)\,dx = \left|\begin{array}{l} x^2 = t \\ 2x\,dx = dt \\ x\,dx = \frac{1}{2}dt \end{array}\right|$$

$$= \int x^3 - 2x^2 + x\,dx + \int \frac{1}{2}\sin(z)\,dz = \frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} + \frac{1}{2}(-\cos z) + C$$

$$F(x) = \frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} - \frac{1}{2}\cos(x^2) + C$$

ZK:

$$F'(x) = x^3 - 2x^2 + x + \frac{1}{2}\sin(x^2)\cdot 2x \quad \checkmark$$

9. $y'' - y = 0$

$y_1 = \sin x$ OK y_2 JE ~~NE~~ ŘEŠENÍ

$y_1' = \cos x$

$y_1'' = -\sin x$

$y_1'' - y_1 = -\sin x - \sin x = 0$ ✓

~~NE~~ ŘEŠENÍ

$y_2(x) = x(x-2)^2 = x(x^2-4x+4) = x^3 - 4x^2 + 4x$

$y_2' = 3x^2 - 8x + 4$

$y_2'' = 6x - 8$

$y_2'' - y_2 = 6x - 8 - x^3 + 4x^2 + 4x = -x^3 + 4x^2 + 10x - 8 \neq 0$

$y_3(x) = e^x$

$y_3' = e^x$ OK

$y_3'' = e^x$

$y_3'' - y = e^x - e^x = 0$

y_3 JE ŘEŠENÍ

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10. $y y' - x = 1$

$y = f(x)$ POČ. POD. $y(0) = 1$

$y' = \frac{dy}{dx}$

$y \cdot \frac{dy}{dx} - x = 1$

NELINEÁRNÍ OBYČEJNÁ DIF. ROVNICE 1. ŘÁDU „NODR1.Ř"

$y \frac{dy}{dx} = 1 + x$

$y\,dy = (1+x)\,dx$

$\int y\,dy = \int (1+x)\,dx$

$\frac{y^2}{2} = x + \frac{x^2}{2} + C$

$y^2 = 2x + x^2 + C$

OBECNÉ ŘEŠENÍ

$y = \pm\sqrt{2x + x^2 + C}$

$y(0) = 1$

$\Rightarrow y = +\sqrt{2x + x^2 + C}$

$1 = \sqrt{0 + 0 + C} \; /^2$

$1 = C$

$y_P = \sqrt{x(x+2)+1} = \sqrt{x^2 + 2x + 1} = \sqrt{(x+1)^2} = |x+1|$

ZK: $2y \cdot y' = 2 + 2x$

$y \cdot y' - x = 1$ ✓