Charged particle motion around magnetized black hole

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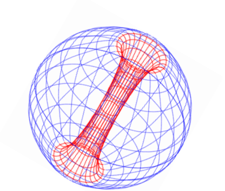
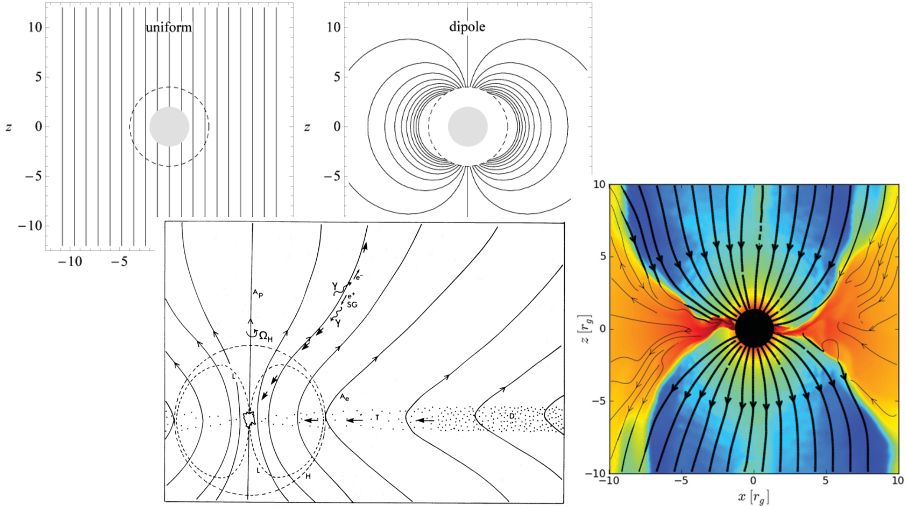


RAGtime workshop #19, 23-26 October, Opava 2017

Black hole magnetosphere

there are only two long-range interactions in physics: **gravity** & electro**magnetism**



lower two figs. stolen from: Blandford, R.D.; Znajek, R.L. *Electromagnetic extraction of energy from Kerr BH* and McKinney J.C., Tchekhovskoy, A.; Blandford, R.D. *General relativistic magnetohydrodynamic simulations of magnetically choked accretion flows around black holes (HARM numerical GRMHD code)*

The stuff around black hole is plasma

An accretion disk is axially symmetric structure formed by plasma in orbital motion around central black hole. Plasma, the fourth state of matter, is:

- In plasma there are free charge carriers.
- Plasma is quasi-neutral.
- Plasma has collective behaviour, it reacts to and also creates electromagnetic fields.

Plasma modeling:

- 3. **N-body problem**
- 2. **Kinetic theory** - distribution function, Boltzmann equation
- 1. **Fluid model** - Magnetohydrodynamics (MHD), fluid, pressure,...
- 0. **Single particle approach** - one particle motion in given magnetic field

⇒ One charged test particle moving in fixed combined magnetic and gravitational field. Gravitational field will be given by Kerr black hole, magnetic field is just uniform mag. field (the simplest one, linear approximation of real mag. field).

⇒ limits of single particle approach:

- macroscopic particle (charged star, small specific charge) - strong mag. field needed
- microscopic particle (electron, large specific charge) - even weak mag. field is OK

  mean free path: $(n \sim 10^{16}\ \mathrm{cm}^{-3}, \mathrm{T} \sim 10^{4}\ \mathrm{K})$ $\qquad \lambda \sim 10^{-3}\ \mathrm{cm}$

  one orbit around black hole: $(M = 10\ \mathrm{M}_{\odot})$ $\qquad l \sim 10^{7}\ \mathrm{cm}$

Equations for charged particle motion

One particle with charge q and mass m in Kerr black hole spacetime with mass M and spin a in the presence of magnetic field can be treated by the **Lorentz equation**

$$\frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} + \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta = \frac{q}{m} g^{\mu\rho} F_{\rho\sigma} u^\sigma$$

where $u^\mu = dx^\mu/d\tau$ is the four-velocity of the particle, $\Gamma^\mu_{\alpha\beta}$ are Christoffel symbols for Kerr black metric (gravity)

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\gamma} \left(g_{\gamma\alpha,\beta} + g_{\gamma\beta,\alpha} - g_{\alpha\beta,\gamma} \right), \quad \mathrm{d}s^2 = g_{tt}\mathrm{d}t^2 + 2g_{t\phi}\mathrm{d}t\mathrm{d}\phi + g_{\phi\phi}\mathrm{d}\phi^2 + \ldots$$

and $F_{\mu\nu}$ is tensor of uniform magnetic field B (mag. field) BH with Wald charge

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad A_\phi = B/2\, g_{\phi\phi}, \quad A_t = B/2\, g_{t\phi} - Ba,$$

Dimensionless quantity $\mathcal{B}$ (magnetic parameter) can be identified as relative Lorenz force:

$$\mathcal{B} = \frac{qBGM}{2mc^4}$$

	electron	proton	Fe+	charged dust
$\mathcal{B} = 0.004$	10^{-5} Gs	0.02 Gs	1 Gs	10^9 Gs

For stellar mass black hole $M \approx 10 M_\odot$, we can have one electron e- in the magnetic field $B = 10^{-5}$ Gs or charged dust grain (one electron lost, $m = 2 \times 10^{-16}$ kg) in field $B = 10^9$ Gs - the absolute value of magnetic field parameter is the same in both cases $\mathcal{B} = 0.004$.

Symmetries, 2D motion, effective potential

- symmetry => conserved quantities: energy $\mathcal{E} = E/m$, angular mome. $\mathcal{L} = L/m$

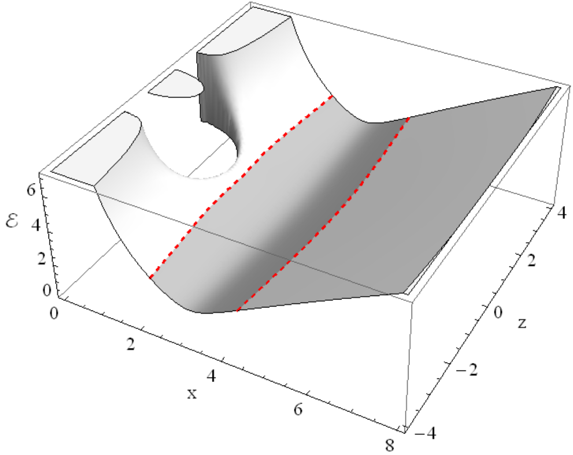
$$-E = \pi_t = g_{tt}p^t + g_{t\phi}p^\phi + qA_t, \quad L = \pi_\phi = g_{\phi\phi}p^\phi + g_{\phi t}p^t + qA_\phi$$

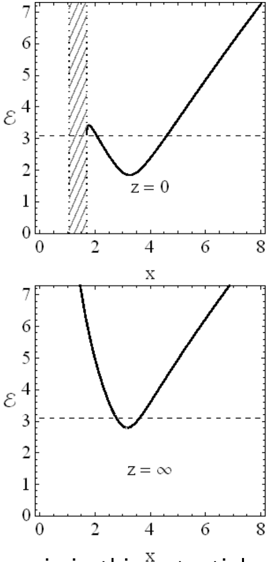
- $t, r, \phi, \theta, u^t, u^r, u^\phi, u^\theta$ + symmetry => r, θ, u^r, u^θ motion 2D effective potential
- condition $g_{\mu\nu}u^\mu u^\nu = -1$ is energetic boundary for particle motion

$$\begin{aligned} V_{\text{eff}}(r,\theta) &= \frac{-\beta + \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}, \\ &\alpha = -g^{tt}, \quad \beta = 2[g^{t\phi}(\mathcal{L} - \tilde{q}A_\phi) - g^{tt}\tilde{q}A_t], \\ &\gamma = -g^{\phi\phi}(\mathcal{L} - \tilde{q}A_\phi)^2 - g^{tt}\tilde{q}^2A_t^2 + 2g^{t\phi}\tilde{q}A_t(\mathcal{L} - \tilde{q}A_\phi) - 1 \end{aligned}$$

- repulsive/attractive Lorentz force, co/contra-rotation => 4 different combinations
  - co-rotating particles $\mathcal{L} > 0$
    - $\mathcal{B} > 0$ (PALO) - Magnetic field lines are oriented in the same direction as the rotation axis of the black hole. The Lorentz force acting on a charged particle corotating with the BH is **repulsive** force is oriented outwards the BH.
    - $\mathcal{B} < 0$ (PLO) - Magnetic field lines are oriented in the opposite direction with respect to the rotation axis of the BH. The Lorentz force acting on a corotating charged particle is **attractive**.
  - contra-rotating $\mathcal{L} < 0$, (RLO $\mathcal{B} > 0$), (RALO $\mathcal{B} < 0$)

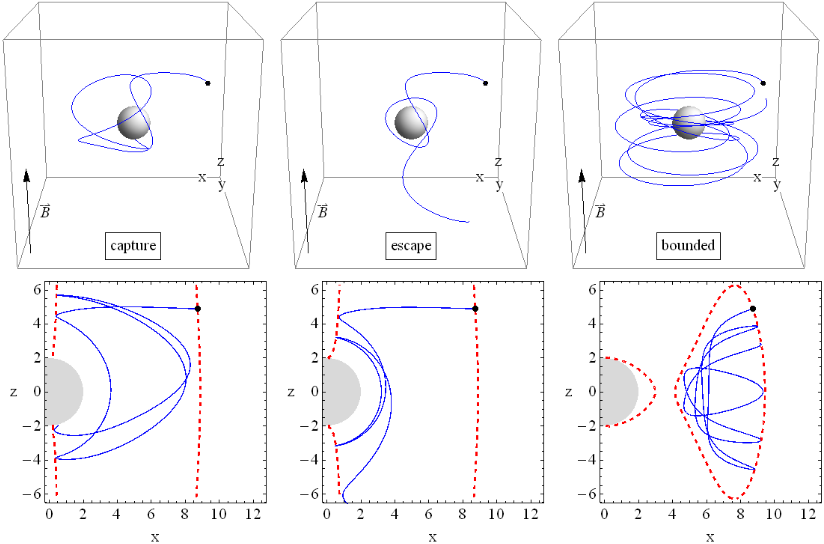
Effective potential $V_{\rm eff}(x,z)$





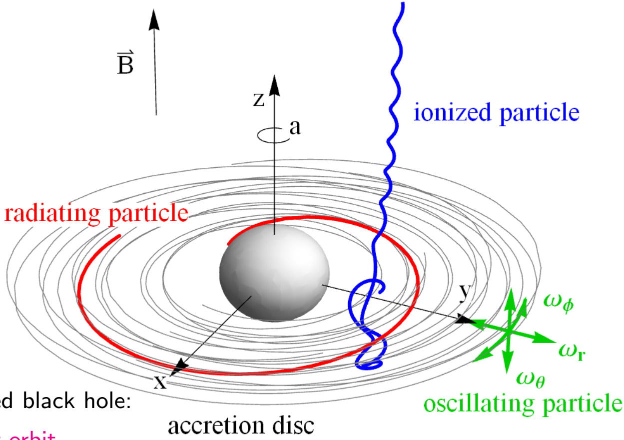
charged particle can escape to infinity along the z axis $\|$ dynamic in this potential is generally chaotic $\|$ $x = r\sin(\theta), z = r\cos(\theta)$

Some examples of charged particle trajectories



Energetic boundary can be open towards BH or infinity; can be closed. Particle can be captured by BH, oscillate around equatorial plane or escape along z-axis.

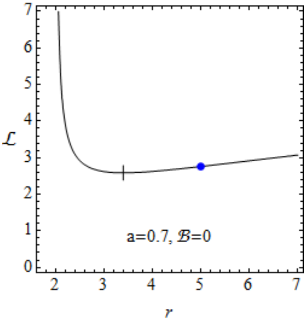
What we've done so far



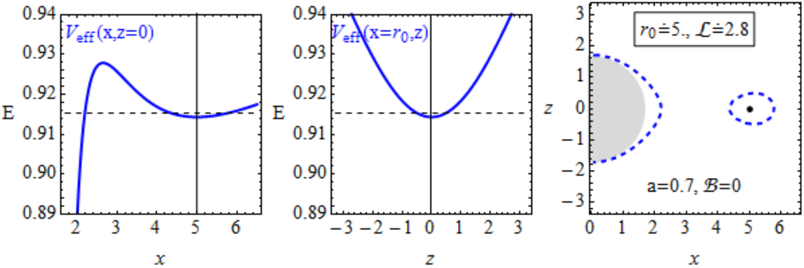
Processes around magnetized black hole:

- (1) Stability of circular orbit
- (2) Charged particle oscillations
- (3) Radiation reaction
- (4) Ionized particle acceleration
- (5) Accretion disc destruction

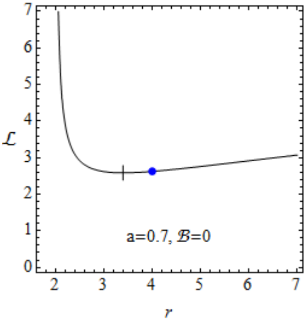
(1) Stability of charged particle circular orbit



- Minima of effective potential $V_{\rm eff}(x,z)$ $\Rightarrow$ stable charged particle circular orbit.
- The sum of all circular orbits forms thin Keplerian accretion disc.
- Angular momenta $\mathcal{L}(r)$ (orbital velocity) for particle on circular orbit (on the left).
- Innermost stable circular orbit (ISCO) is accretion disc inner edge.



(1) Stability of charged particle circular orbit



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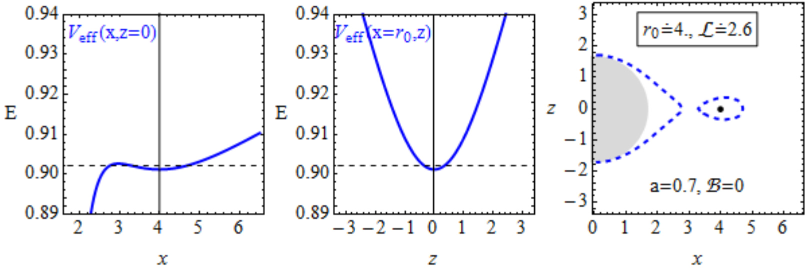

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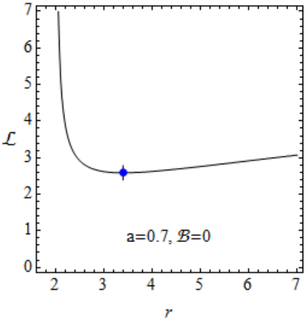

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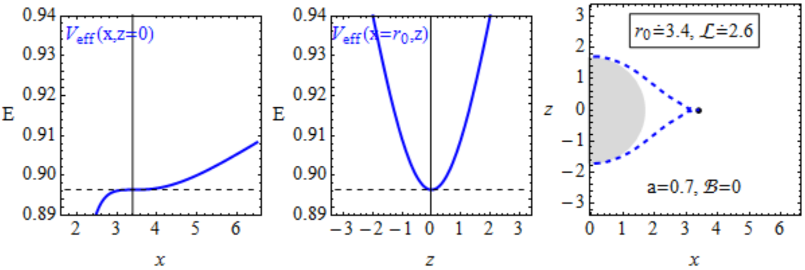
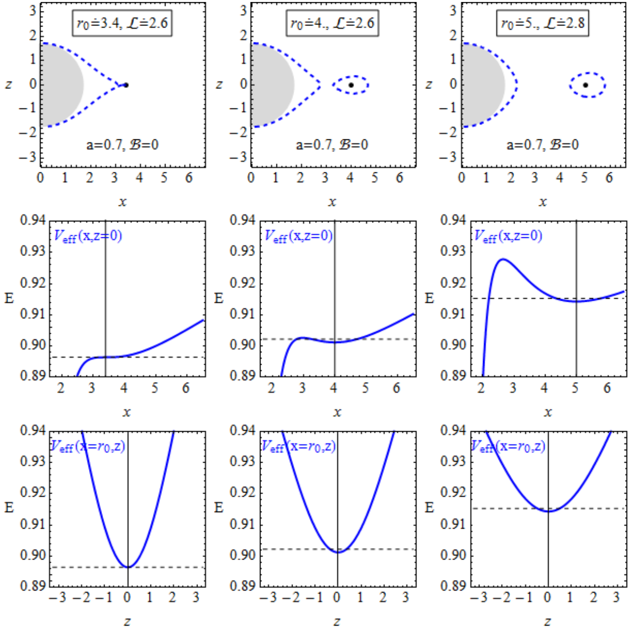
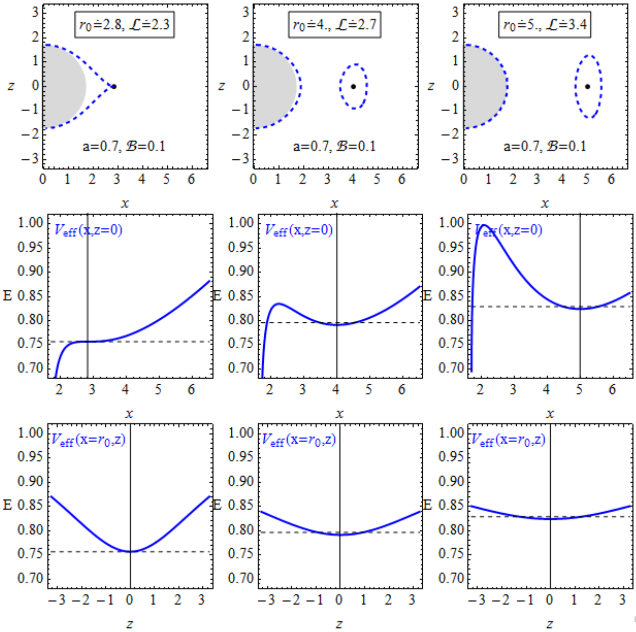


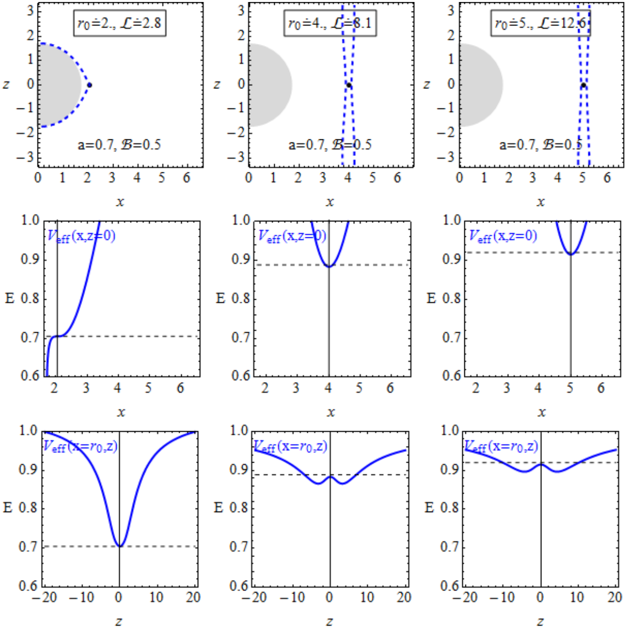
(1) Stability of circular orbit $\mathcal{B} = 0$



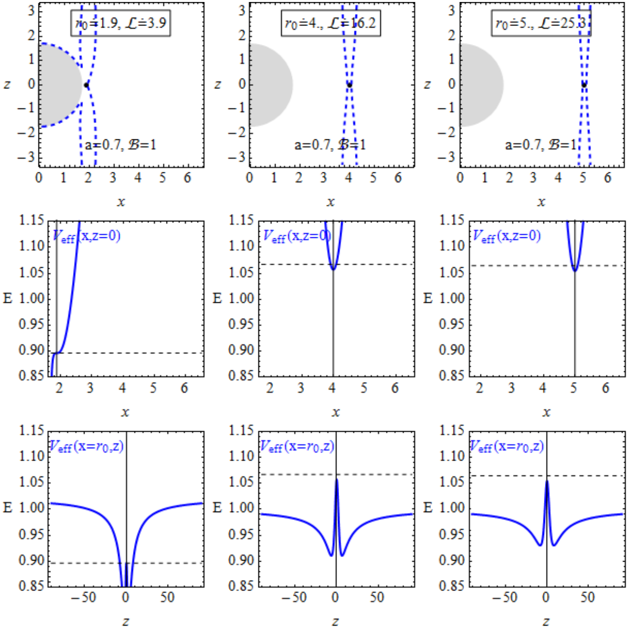
(1) Stability of circular orbit $\mathcal{B} = 0.1$



(1) Stability of circular orbit $\mathcal{B} = 0.5$

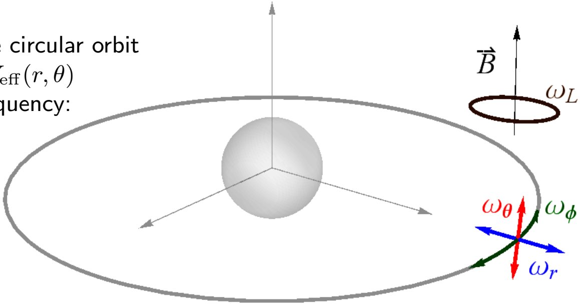


(1) Stability of circular orbit $\mathcal{B} = 1$



(2) Magnetic field and microquasar QPOs

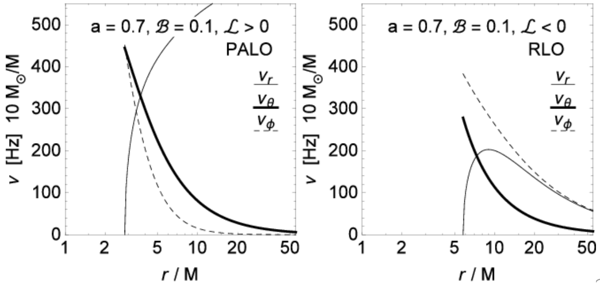
Perturbation of particle circular orbit located at minima of $V_{\rm eff}(r,\theta)$ will give us angular frequency:
Keplerian ω_ϕ,
vertical ω_θ,
Larmor $\omega_{\rm L}$,
horizontal (radial) $\omega_{\rm r}$



frequency measured:

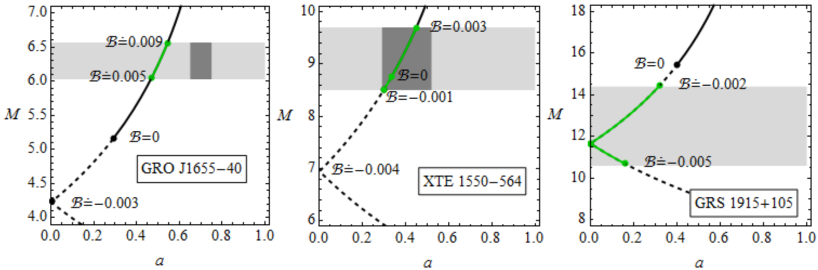
- ω_α locally
- Ω_α at infinity (redshifted)
- ν_α in physical units

$$\nu_\alpha = \frac{1}{2\pi}\frac{c^3}{GM}\,\Omega_\alpha[\mathrm{Hz}],$$



(2) Magnetic field and microquasar QPOs

Quasi-periodic oscillations (QPOs) of the X-ray power density are observed in microquasars. The QPOs are sometimes detected with the twin peaks (upper f_{U} and lower f_{L}) which have frequency ratio close to $3:2$. We will examine magnetic field influence on the QPOs phenomena, determining mass and spin of the black hole inside the GRS 1915+105, XTE 1550-564 and GRO 1655-40 sources.



The results are presented for different strength of magnetic field $\mathcal{B}$ (thick curves) - they are compared with Kerr black hole mass M and spin a obtained by different independent method (grey rectangles).

• M. Kološ, A. Tursunov and Z. Stuchlík, *Possible signature of magnetic field in microquasar QPOs*, submitted to EPJc (2017), [**arXiv:1707.02224**]

(3) Radiation reaction of a charged particle

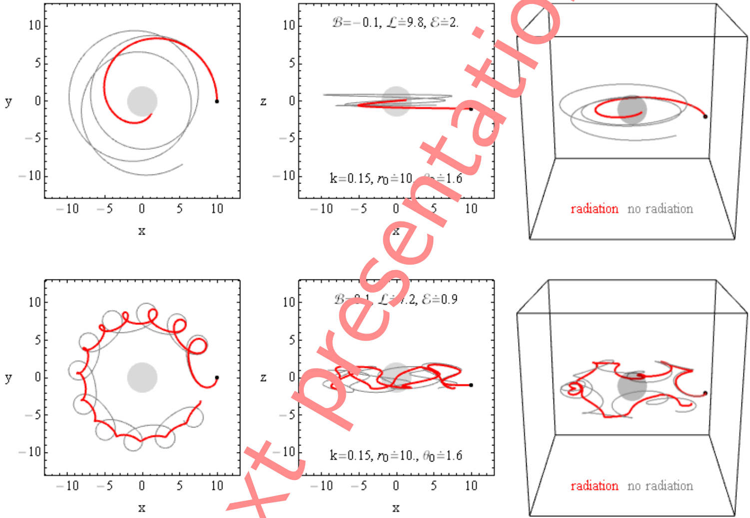
Synchrotron radiation emitted by a charged particle leads to appearance of the **back-reaction force** which can significantly affect its motion. We study the dynamics of a charged particle undergoing radiation reaction force in combined Schwarzschild black hole gravitational filed and an external asymptotically uniform magnetic field

$$\frac{\mathrm{d}u^{\mu}}{\mathrm{d}\tau} + \Gamma^{\mu}_{\alpha\beta}u^{\alpha}u^{\beta} = \frac{q}{m}F^{\mu}{}_{\nu}u^{\nu} + \frac{q}{m}\mathcal{F}^{\mu}{}_{\nu}u^{\nu},$$

where the first term on the right hand side corresponds to the Lorentz force with $F_{\mu\nu}$, while the second term with $\mathcal{F}_{\mu\nu}$ is the self-force of charged particle with the radiative field.

• A. Tursunov, M. Kološ, Z. Stuchlík and D. V. Gal'tsov : *Radiation reaction of a charged particle orbiting a weakly magnetized Schwarzschild black hole*, in preparation

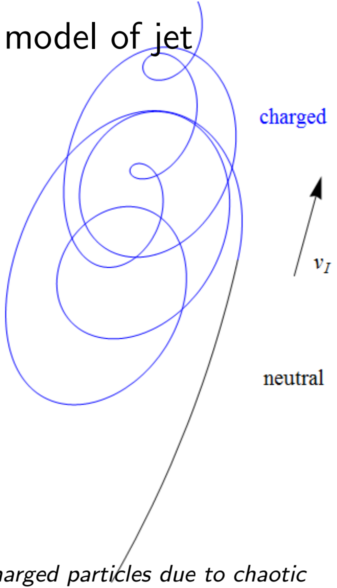
(3) Radiation reaction of a charged particle



Depending on the orientation of the Lorentz force, the oscillating charged particle either **spirals down** to the black hole ($\mathcal{B} < 0$ - first line of figure), or stabilizes the circular orbit by **decaying its oscillations** ($\mathcal{B} > 0$ - second line of figures).

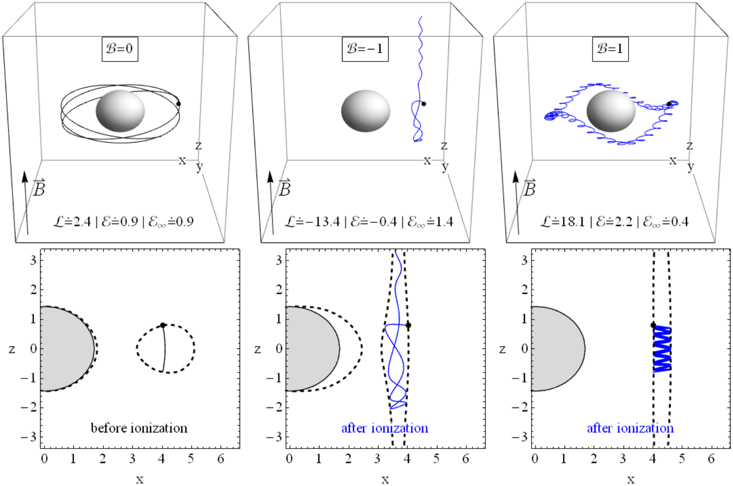
(4) Ionized particle acceleration - model of jet

The neutral particles forming accretion disk can get ionized and hence start to feel the magnetic field. **Chaotic scattering** in combined black hole gravitational and uniform magnetic field then occur and interchange between velocity around black hole u^{ϕ} and velocity along rotational axis u^{z} provides mechanism for charged particle **acceleration and escape** along the magnetic field lines.
Ionization process (particle collisions, irradiation) changeling only particle charge - charged particle initial velocity is given by the velocity before ionization $v_{\mathrm{II}}(0) = v_{\mathrm{I}}(0)$ - mechanical momenta are conserve during ionization



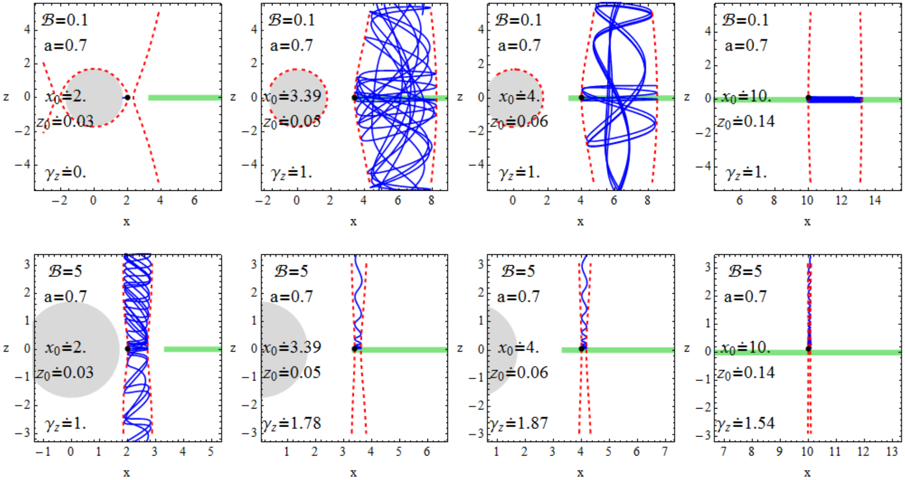
• Z. Stuchlík and M. Kološ: *Acceleration of the charged particles due to chaotic scattering in the combined black hole gravitational field and asymptotically uniform magnetic field*, EPJc 76 (1), 1-21 (2016), [**arXiv:1511.02936**].

(4) Ionized particle acceleration - model of jet



Neutral particles from the accretion disc are located on a spherical orbit (r =cost.) **(1)**. They get ionized and escape to the infinity along magnetic field lines with relativistic velocities **(2)**, or they can just stay and oscillate in equatorial plane **(3)** (depends on initial conditions).

(5) Accretion disc destruction



Fate of ionized particles from the accretion disc (depends on radial position r, mag. field $\mathcal{B}$, BH rotation a)

- some particles will oscillate, some will escape, some just dont want to fall into BH.

Summary & What to do?

We have studied processes around magnetized black hole:

- (1) Stability of circular orbit - charge particle ISCO come close to the BH horizont, instability in vertical z direction
- (2) Charged particle oscillations - charge particle frequencies can be well related to the observed microquasar QPOs
- (3) Radiation reaction - see presentation of Arman Tursunov
- (4) Ionized particle acceleration - charged particles are escaping along magnetic field lines in vertical z direction, relativistic escape velocity even for small magnitude of magnetic field $\mathcal{B} \sim 1$ and moderate black hole spin $a \sim 0.5$
- (5) Accretion disc destruction - particles from the neutral accretion disc will be after ionisation: escaping/oscillating/falling into BH

Future tasks

- Single particle approach: accretion disc destruction, radiation reaction in Kerr...
- Single particle is not enough $\Rightarrow$ MHD!

Thank you for your attention